Killara HIGH SCHOOL

Student Number

2018
TRIAL EXAMINATION

Extension 2 Mathematics

General Instructions
- Reading time – 5 minutes
- Working tine - 3 hours
- Write using blue or black pen
  Black pen is preferred
- Approved calculators may be used
- A formulae sheet is provided separately
- In Questions 11-16 show relevant mathematical reasoning and/or calculations
- **Start a new booklet for each question**

Total Marks – 100

Section I - Pages 3-7
10 marks
- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II - Pages 8-16
90 marks
- Attempt Questions 11 – 16
- Allow about 2 hour and 45 minutes for this section

Question	Marks
1 - 10	/10
11	/15
12	/15
13	/15
14	/15
15	/15
16	/15
Total	/100

THIS QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

This assessment task constitutes 40% of the Higher School Certificate Course Assessment

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section
Use the multiple-choice answer sheet for questions 1 – 10 (Detach from paper)

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1. The sum of eccentricities of two different conics is 2.5. Which pair of conics could this be?

   (A) Circle and ellipse

   (B) Ellipse and parabola

   (C) Parabola and hyperbola

   (D) Rectangular hyperbola and circle

2. What value of z satisfies $z^2 = 9 - 40i$?

   (A) $-5-4i$

   (B) $5-4i$

   (C) $4-5i$

   (D) $-4-5i$

3. Given α, β, γ are roots of $x^3 + nx^2 - px - k = 0$.

   Find the value of $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$

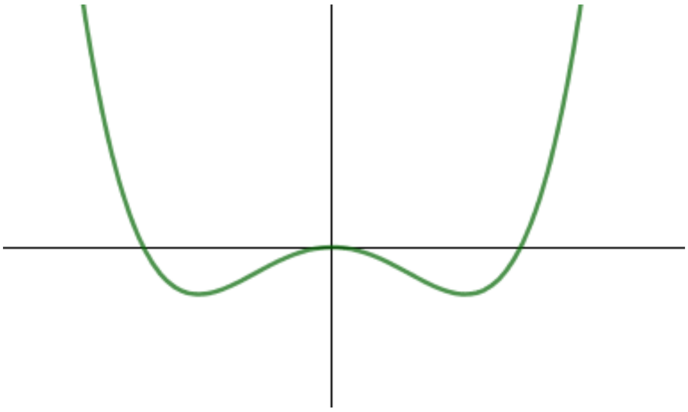
   (A) 0

   (B) $\frac{p}{k}$

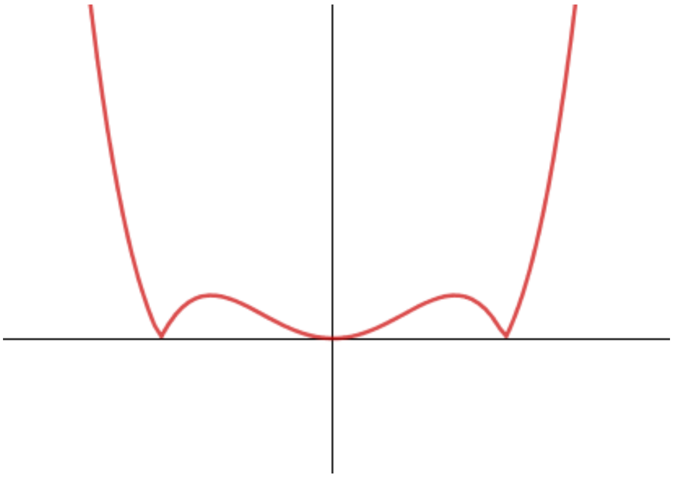
   (C) $-\frac{p}{k}$

   (D) $-\frac{1}{n}$

4. The graph of the function $y = f(x)$ is shown.



A second graph is obtained from the function $y = f(x)$



Which equation best represents the second graph?

(A) $y^2 = f(x)$

(B) $y = |f(x)|$

(C) $y = f(|x|)$

(D) $y^2 = |f(x)|$

5. If $z = -1 + \sqrt{3}i$ which expression is equal to z^5 ?

(A) $32cis\left(-\frac{\pi}{3}\right)$

(B) $2cis\left(-\frac{2\pi}{3}\right)$

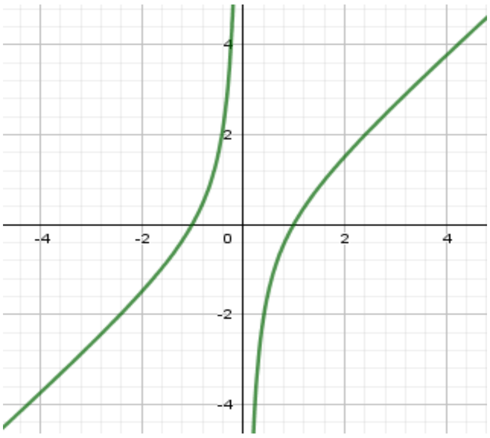
(C) $32cis\left(-\frac{2\pi}{3}\right)$

(D) $2cis\left(-\frac{\pi}{3}\right)$

6. Let $f(x) = \frac{x^k + a}{x}$ where k and a are real constants.

If k is an odd integer which is greater than 1 and $a < 0$, a possible graph of $f(x)$ could be:

A



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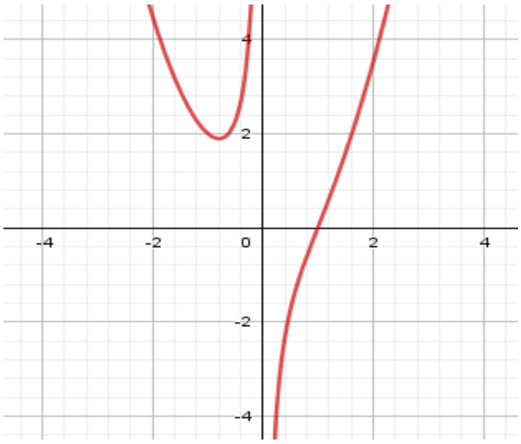

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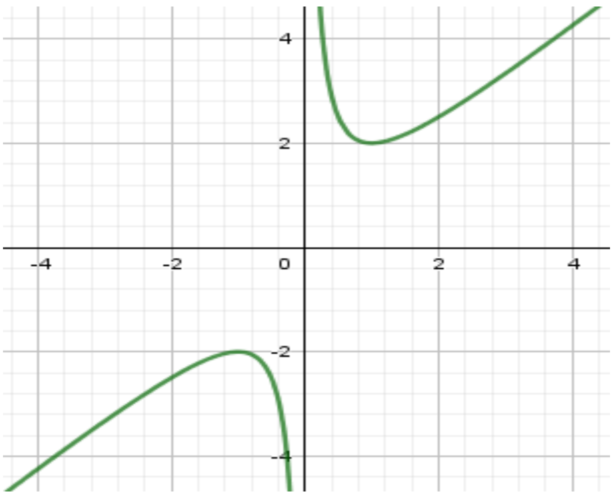

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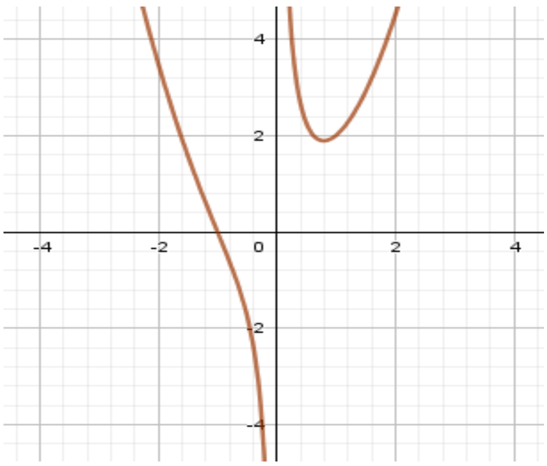
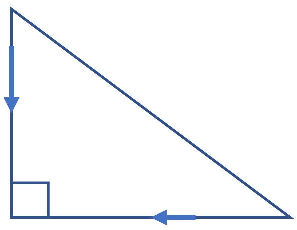


7. A particle P of mass $5kg$ is subject to forces 12 Newtons and 9 Newtons acting in perpendicular directions. The magnitude of the acceleration of the particle in ms^{-2}, is

(A) 3
(B) 4.2
(C) 15
(D) 75



9 N

8. A particle moving in a Simple Harmonic Motion oscillates about a fixed point O in a straight line with a period of 10 seconds. The maximum displacement of P from O is 5 m. Which of the following statement/s is/are true?

If P is at O moving to the right, then 22 seconds later P will be:

I. Moving towards O
II. Moving with a decreasing speed
III. At a distance $5sin(2\pi/5)\ m$ to the right of O

P

(A) I, II and III
(B) I and II only
(C) II and III only
(D) None of the above

9. z is a complex number such that $z = \left(1 - \sqrt{a}\, sin\ t\right) + i(1 - \frac{1}{b}\cos t)$, where $t \geq 0, a$ and b are positive real numbers.

The locus of z on an Argand diagram will always be a circle if:

(A) $ab^2 = 1$

(B) $a^2b = 1$

(C) $ab^2 \neq 1$

(D) $a^2b \neq 1$

10. Suppose q, r, s and t are positive real numbers. Which of the following is the correct expression?

(A) $\int \frac{px+q}{rx+s}\, dx = \frac{p}{r}\left[x - \left(\frac{q}{p} + \frac{s}{r}\right) \ln\,(rx+s)\right] + C$

(B) $\int \frac{px+q}{rx+s}\, dx = \frac{p}{r}\left[x + \left(\frac{q}{p} + \frac{s}{r}\right) \ln\,(rx+s)\right] + C$

(C) $\int \frac{px+q}{rx+s}\, dx = \frac{p}{r}\left[x + \left(\frac{q}{p} - \frac{s}{r}\right) \ln\,(rx+s)\right] + C$

(D) $\int \frac{px+q}{rx+s}\, dx = \frac{p}{r}\left[x - \left(\frac{q}{p} + \frac{s}{r}\right) \ln\,(rx+s)\right] + C$

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11(15 marks) Use a SEPARATE writing booklet

a) Express $\dfrac{2+5i}{3-i}$ in the form $x+iy$ where x and y are real. 2

b) Consider the complex numbers $z=3-2i,\ w=-2+i$

(i) Express $z+w$ in modulus argument form 2

(ii) write down $\bar{z}+\bar{w}$ in modulus argument form 1

c) Given $z=4cis\dfrac{\pi}{3},\ w=2cis\dfrac{5\pi}{6}$

(i) Calculate $z.w$ in modulus argument form 1

(ii) Convert $z.w$ to cartesian form 1

d) Express $\dfrac{3x+2}{(x+1)(x+2)^2}$ in partial fractions 2

e) Find the value of $\dfrac{dy}{dx}$ at the point $(5,-3)$ on the curve $x^2+4yx+5y^2=10$ 3

f) Using the substitution $t=\tan\dfrac{x}{2}$ find $\displaystyle\int\frac{1}{4\sin x+3\cos x}dx$, leaving your final answer in terms of t. 3

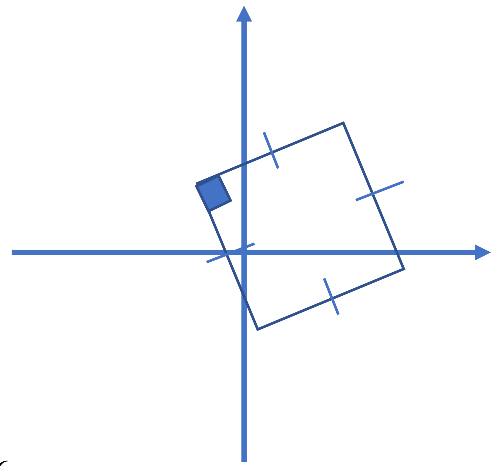
End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet

a) Sketch the region in the Argand diagram where $|z+1+i| \le 1$ and $-\pi \le \arg z \le -\dfrac{3\pi}{4}$ 2

b) The points A,B,C,D represent the complex numbers a,b,c and d respectively. The points form a square as shown on the diagram below. 2

By using vectors or otherwise, show that $b = c(1+i) - id$



c) Factorise $z^4 + z^2 - 6$

(i) over the irrational number field 1

(ii) over the complex field and list the complex roots 1

d) (i) By writing $f(x) = \dfrac{(x+4)(x+3)}{(x-1)}$ in the form $f(x) = mx + b + \dfrac{a}{x-1}$ find the equation of the oblique asymptote of $f(x) = \dfrac{(x+4)(x+3)}{(x-1)}$ 2

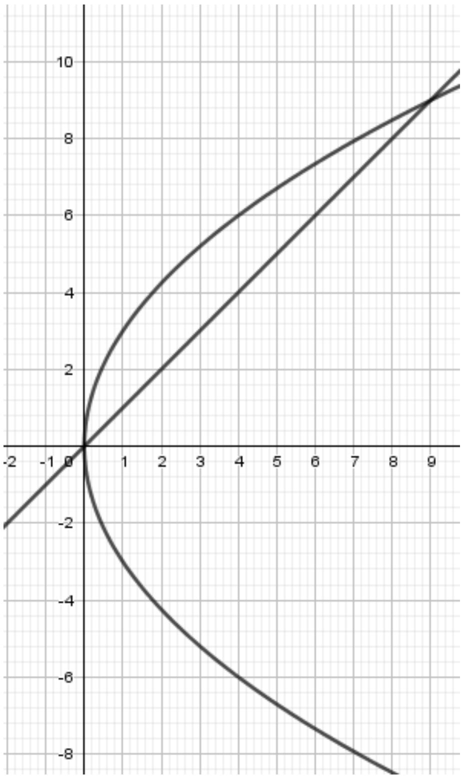
(ii) Sketch $f(x) = \dfrac{(x+4)(x+3)}{(x-1)}$ clearly indicating all intercepts and asymptotes 2

(iii) from your graph of $f(x)$ draw a sketch of $y = \sqrt{f(x)}$ 2

(iv) draw a half to a third of a page sketch of $y^2 = f(x)$ 1

Question 12 continues on the next page

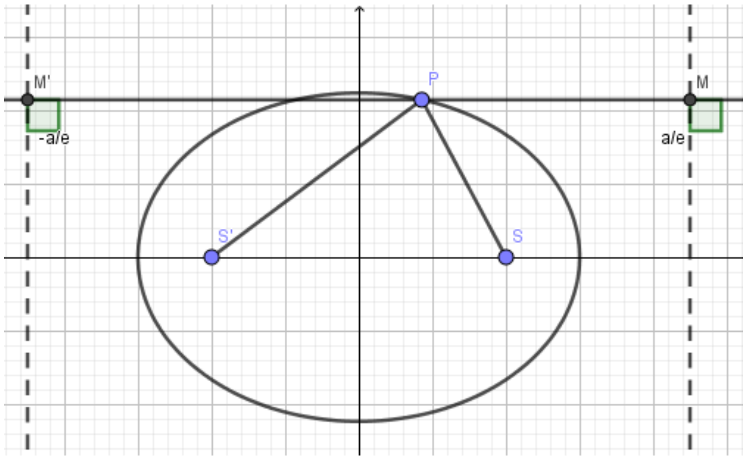
e) Find the volume of the solid generated by rotating the region bounded by $y^2 = 9x$ and $y = x$ about the y-axis using the method of cylindrical shells **2**



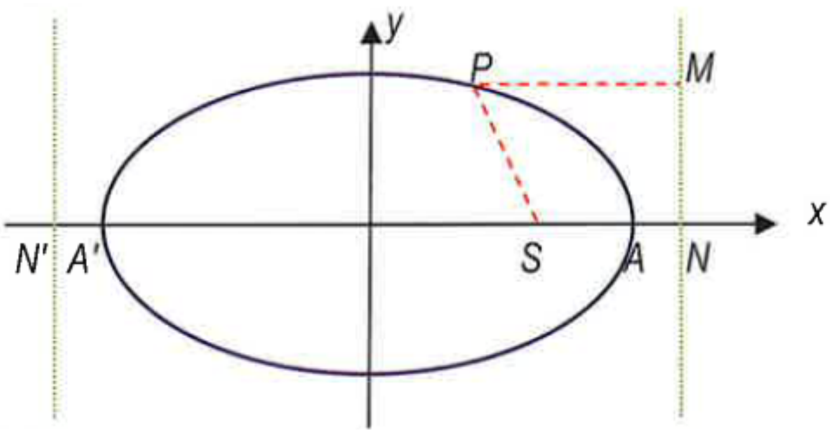
End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet

a) Prove the sums of the focal distances from a point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is equal to *2a*. You may refer to the diagram below. **2**



b) By referring to the diagram below, taking the coordinates of A as $(a,0)$, and A' as $(-a,0)$. *Using the definition of an ellipse and* $\frac{SP}{PM}=e$:

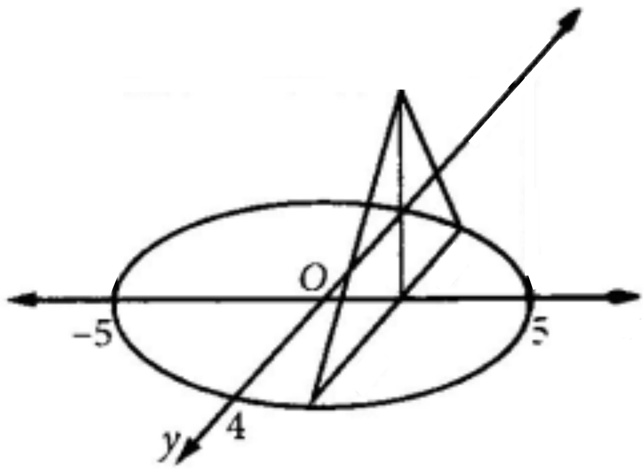


(i) Prove the positive directrix equation is $x=\frac{a}{e}$ **2**

(ii) Prove that the focus S has coordinates $(ae,0)$ **1**

(iii) Hence, prove the equation of an ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ **2**

c) A solid has a base in the form of an ellipse with major axis 10 units and minor axis 8 units, as shown in the diagram below. Find the volume of the solid if every section perpendicular to the major axis is an equilateral triangle. 4



d) A concrete crushing plant turns concrete waste into fine gravel. The gravel pours off a conveyor belt at the rate of $12m^3 / min$. The falling gravel forms a pile in the shape of a cone on the ground (*Note*: you can assume that the plant operator shuts down the crusher when the top of the cone nears the conveyor belt). The base of the cone is always equal to 1.25 times the height of the cone.

(i) Show that when the height is *h* metres, the volume $V\ m^3$ of gravel is given by 1

$$V = \frac{25\pi h^3}{192}$$

(ii) Hence determine how fast the height of the pile is increasing (in *m*/*min*) when the gravel pile is 2 metres high. 3

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet

a) (i) Find the exact value of $\int_0^{\frac{\pi}{6}} 2\ cosec4x tan2x\ dx$ 2

(ii) Use a suitable substitution or otherwise to show that 3

$$\int_0^k \frac{x^2}{\sqrt{1-4x^2}}\ dx = \frac{1}{16}\left[sin^{-1}2k - 2k\sqrt{1-4k^2}\right],$$

where k is a real number.

b) If α and β are the roots of $x^2 + px + q = 0$. If $S_n = \alpha^n + \beta^n$, it can be shown that $S_{2n} = {S_n}^2 - 2q^n$ and $S_{2n+1} = S_n S_{n+1} + pq^n$ 3
(You do NOT need to prove this)

Express S_7 in terms of p and q.

c) (i) Find real numbers a and b such that 2

$$\frac{5-x}{(2x+3)(x^2+1)} = \frac{a}{2x+3} - \frac{bx-1}{x^2+1}$$

(ii) Hence find $\int \frac{5-x}{(2x+3)(x^2+1)} dx$ 2

d) If $I_n = \int \frac{\cos(2nx)}{\cos x}\ dx$, show that $I_n = \frac{2\sin(2n-1)x}{(2n-1)} - I_{n-1}$. 3

End of Question 14

Question 15 (15 marks) Use a SEPARATE writing booklet

a) (i) Differentiate $y = sin^{-1}(x) + cos^{-1}(x)$, with respect to x. **1**

(ii) Evaluate $cos^{-1}(x) + cos^{-1}(-x)$. **1**

(iii) Hence solve $\sin^{-1}(x) + \tan^{-1}\left(\frac{5x}{2x^2+2}\right) = cos^{-1}(-x) - \frac{\pi}{4}$ **2**

b) Consider $f(x) = x - \ln\left(1 + x + \frac{x^2}{2}\right)$

(i) Show that $f(x)$ is an increasing function. **2**

(ii) Hence show that $e^x > 1 + x + \frac{x^2}{2}$ for $x > 0$. **2**

c) A particle of mass $m\ kg$ is falling from rest, experiences air resistance of mkv^2 Newtons, where k is a positive constant and $v\ ms^{-1}$is the velocity of the particle. Acceleration of gravity is $g\ ms^{-2}$.

(i) Draw the force diagram to show that the equation of motion of the particle is $\ddot{x} = g - kv^2$, where x metres is the distance the particle fell from its original position. **1**

(ii) Explain how the value of the terminal velocity, $V\ ms^{-1}$, of the particle be obtained and state its value in terms of k and g. **1**

(iii) Show that the velocity of the particle, $v\ ms^{-1}$, at t seconds is given by **3**

$$v = V\left[\frac{e^{2kVt} - 1}{e^{2kVt} + 1}\right]$$

(iv) Show that the position of the particle, x metres, in terms of v, is given by $x = \frac{1}{2k} ln\left[\frac{g}{g-kv^2}\right]$ **2**

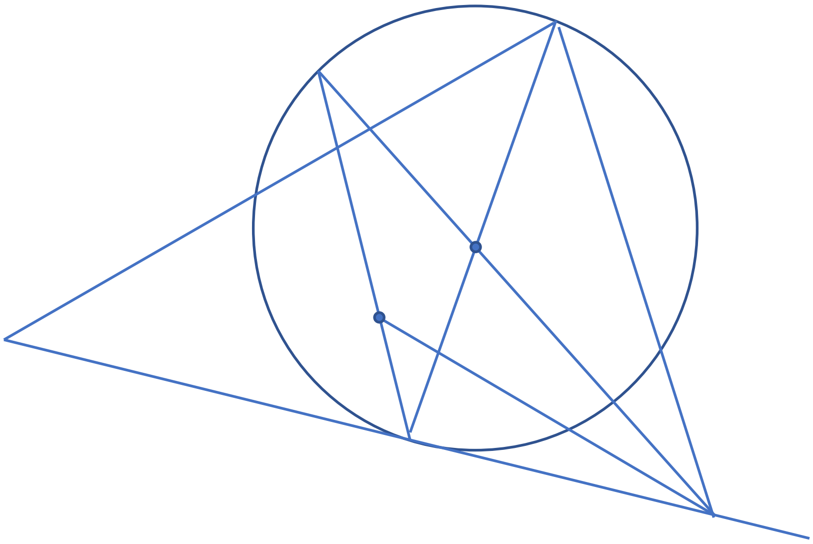
End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet

a) In the diagram below, AC is the diameter of circle $AECFG$ with centre O.

BD is the tangent to the circle at C.

H is a point on GC such that $\angle BHC = \frac{\angle GHB}{3}$



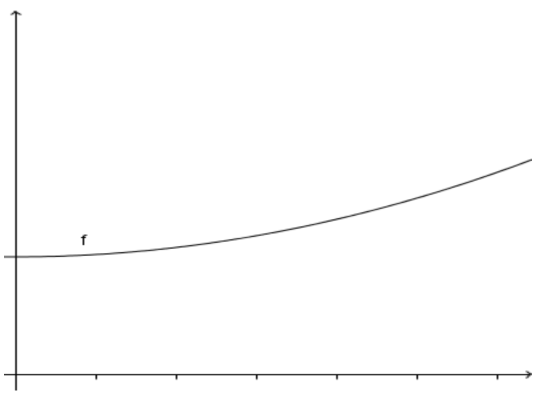
(i) Prove that $DFEB$ is a cyclic quadrilateral. **3**
Tip: you may wish to add a line (construction).

(ii) Prove that $\angle HBG = \angle HBC$ **2**

b) (i) Given that f is a continuous function as shown below, explain, with aid of a sketch, why the value of **3**

F

$\lim\limits_{n\to\infty} \frac{1}{n}\left\{f\left(\frac{1}{n}\right) + f\left(\frac{2}{n}\right) + f\left(\frac{3}{n}\right) + ---- + f\left(\frac{n}{n}\right)\right\}$ is $\int_0^1 f(x)dx$.

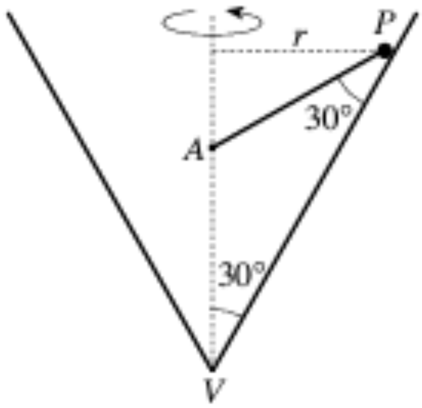


(ii) Hence evaluate $\lim\limits_{n\to\infty} \frac{1}{n}\left(\frac{1+\sqrt[3]{2}+\sqrt[3]{3}+ ----- + \sqrt[3]{n}}{\sqrt[3]{n}}\right)$ **2**

D

Question 16 continues on the next page

c)



A hollow circular cone is fixed with its axis vertical and it vertex *V* downwards. A particle *P*, of mass $m\ kg$, is attached to a fixed point *A* on the axis of the cone by means of a light inextensible string of length equal to *AV* metres. The particle moves with constant speed *v m/s* in a horizontal circle on the smooth inner surface of the cone, with the string taut. The radius of the circle is *r* metres, and angles *APV* and *AVP* are each 30° (see diagram)

i) Find the tension in the string in terms of *m, g, v* and *r*. **3**

ii) Deduce that $\frac{v^2}{gr} > \sqrt{3}$. **2**

End of Question 16

End of Paper

Extension 2 - KHS - 2018
Trial - Solutions with feedback.

M/choice

The sum of eccentricities ...

Q1

(C) Parabola and Hyperbola

Parabola $e=1$
Hyperbola $e>1$

Q2 what value of z satisfies $z^2 = 9-40i$

$(x+iy)^2 = 9-40i$
$x^2-y^2 = 9$
$2ixy = -40i \Rightarrow xy = -20 \rightarrow y = \frac{-20}{x}$

$x^2+y^2 = 41$ +
$x^2-y^2 = 9$
$2x^2 = 50 \rightarrow x^2 = 25 \rightarrow x = \pm 5$
$x=5$ $y=-4$
$x=-5, y=4$
$\therefore$ (B) $5-4i$

Q3 Given α, β, γ are roots of $x^3+nx^2-px-k=0$

find $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$

$$\frac{\alpha\beta+\alpha\gamma+\beta\gamma}{\alpha\beta\gamma}$$

$$\alpha+\beta+\gamma = -\frac{b}{a} = -n$$

$$\Sigma\alpha\beta = \frac{c}{a} = -p$$

$$\Pi\alpha = -\frac{d}{a} = --k = k$$

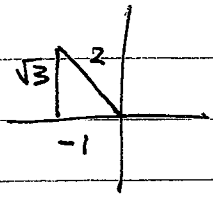
$$\therefore = \frac{-p}{k} \quad (C)$$

4 The graph of $y=f(x)$ is shown ...

$y = |f(x)|$ (B)

- students should realise quickly as section below x-axis reflected above x-axis.

5 If $z = -1 + \sqrt{3}\,i$ which expression equals z^5



$\sin\theta = \frac{\sqrt{3}}{2} \quad \cos\theta = \frac{-1}{2}$

$\therefore z = 2 \operatorname{cis} \frac{2\pi}{3}$

$z^5 = 2^5 \operatorname{cis} \frac{2\pi}{3} \times 5$ By De Moivre's

$= 32 \operatorname{cis} \frac{10\pi}{3}$

$= 32 \operatorname{cis}\left(\frac{-2\pi}{3}\right)$ (C)

6. $f(x) = \dfrac{x^k + a}{x}$

$k > 1$, $a < 0$

V.A $x = 0$

e.g $\dfrac{x^3 - 1}{x}$

$x \overline{) x^3 - 1}$ quotient x^2

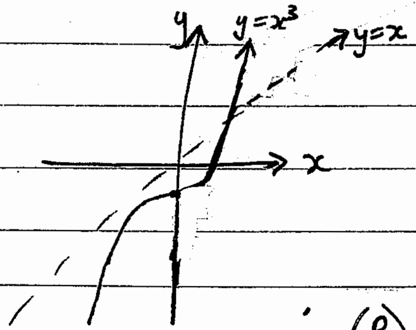
x^3

-1

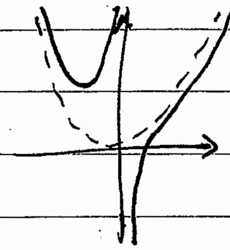
$y = x^2 - \dfrac{1}{x}$

as $x \to \infty$

$y \to x^2$

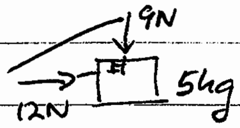


$\Longrightarrow$



$\therefore$ (B)

7.



By Pythagoras' $F = \sqrt{9^2 + 12^2}$

$F = 15$

$f = ma$

$15 = 5a$

$\therefore a = 3$ (A)

Q8

Period = 10 seconds

$\therefore \frac{2\pi}{n} = 10 \Rightarrow n = \frac{\pi}{5}$

Max^{m} displacement = amplitude = $a = 5$ m

$x = 0, t = 0 \Rightarrow x = 5\sin\left(\frac{\pi t}{5}\right)$

when $t = 22$ $\quad x = 5\sin\left(22\frac{\pi}{5}\right)$

$= 5\sin\left(4\pi + \frac{2\pi}{5}\right)$

$= 5\sin\left(\frac{2\pi}{5}\right)$ ——— (III) ✓

$\ddot{x} = -\frac{\pi}{5}\sin\left(\frac{2\pi}{5}\right) \Rightarrow$ decreasing speed —— (II) ✓

moving to the right of O $\Rightarrow$ (I) ✗

$\therefore$ The choice is (B)

9

$x = 1 - \sqrt{a}\sin t$

$\sqrt{a}\sin t = 1 - x$

$\sin t = \frac{1-x}{\sqrt{a}}$

$y = 1 - \frac{1}{b}\cos t$

$\frac{1}{b}\cos t = 1 - y$

$\cos t = b(1-y)$

$\sin^2 t + \cos^2 t = 1$

$\left(\frac{1-x}{\sqrt{a}}\right)^2 + [b(1-y)]^2 = 1$

$\left\{ \frac{(1-x)^2}{a} + b^2(1-y)^2 = 1 \right\} \times a$

$1(1-x)^2 + ab^2(1-y^2) = a$

to be a circle, coefficient of $(1-x)^2$ and $(1-y^2)$ must be the same

$\therefore ab^2 = 1$ (A)

10 $\dfrac{px+q}{rx+s}$

$$rx+s \,\overline{)\,px+q\,}\quad \frac{p}{r}$$
$$px+\frac{ps}{r}\ -$$
$$q-\frac{ps}{r}$$

$$\int \frac{px+q}{rx+s}\,dx$$

$$= \int\left(\frac{p}{r} + \frac{q-\frac{ps}{r}}{rx+s}\right)dx$$

$$= \int\left(\frac{p}{r} + \frac{qr-ps}{r(rx+s)}\right)dx$$

for $\Rightarrow \displaystyle\int \frac{qr-ps}{r(rx+s)}\,dx$

$$= \frac{p}{r}\int \frac{\frac{qr}{p}-s}{rx+s}$$

$\begin{cases} y = rx+s \\ y' = r \end{cases}$

$$= \frac{p}{r}\int \frac{\frac{qr-sp}{p}}{rx+s}\,dx$$

$$= \frac{p}{r} \times \frac{qr-sp}{pr} \cdot \int \frac{r}{rx+s}\,dx$$

$\therefore$ (c)

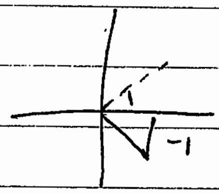
11 a $$\frac{2+5i}{3-i} \times \frac{3+i}{3+i} = \frac{6+15i+2i-5}{9-i^2} \quad [1 \text{ mark}]$$

$$= \frac{1+17i}{10}$$

$$= \frac{1}{10} + \frac{17}{10}i \quad [1 \text{ mark}]$$

b i $z = 3-2i$, $w = -2+i$

$z+w = 1-i$



$\sqrt{2}$ [1 mark]

$\frac{-\pi}{4}$ [1 mark]

$$\therefore z+w = \sqrt{2} \operatorname{cis}\left(\frac{-\pi}{4}\right)$$

ii $\bar{z}+\bar{w}$ → from diagram above $= \sqrt{2} \operatorname{cis} \frac{\pi}{4}$ [1 mark]

OR

$$3+2i + -2-i = 1+i \Rightarrow \sqrt{2} \operatorname{cis}\left(\frac{\pi}{4}\right)$$

c i $$4\operatorname{cis}\frac{\pi}{3} \times 2\operatorname{cis}\frac{5\pi}{6} = 8\operatorname{cis}\frac{7\pi}{6} = 8\operatorname{cis}\left(\frac{-5\pi}{6}\right) \quad [1 \text{ mark}]$$

ii $$8\left(\cos\left[\frac{-5\pi}{6}\right] + i\sin\left[\frac{-5\pi}{6}\right]\right)$$

$$= 8\left(\frac{-\sqrt{3}}{2} - \frac{i}{2}\right)$$

$$= -4\sqrt{3} - 4i \quad [1 \text{ mark}]$$

11d

$$\frac{3x+2}{(x+1)(x+2)^2} = \frac{a}{x+1} + \frac{b}{x+2} + \frac{c}{(x+2)^2}$$

$$\therefore\ 3x+2 = a(x+2)^2 + b(x+1)(x+2) + c(x+1)$$

sub. $x=-1 \rightarrow \underline{-1=a}$

sub $x=-2 \rightarrow -4=-c \rightarrow \underline{c=4}$

coeff. of x^2 $\quad 0x^2 = ax^2 + bx^2$

$\Rightarrow \quad 0 = a+b$

$\therefore\ \underline{b=1}$

$$\therefore\ \frac{3x+2}{(x+1)(x+2)^2} = \frac{-1}{x+1} + \frac{1}{x+2} + \frac{4}{(x+2)^2}$$

[2 marks a, b, c correct]
[1 mark 2 pronumerals correct]
or correct process with minor error.

also paid $\dfrac{-1}{x+1} + \dfrac{x+6}{(x+2)^2}$

11e $x^2 + 4yx + 5y^2 = 10$

$$2x + 4\frac{dy}{dx}\cdot x + 4y\cdot 1 + 10y\frac{dy}{dx} = 0$$ [1 mark]

$$\frac{dy}{dx}(10y+4x) = -2x-4y$$

$$\frac{dy}{dx} = \frac{-(2x+4y)}{10y+4x}$$

$$= \frac{-\cancel{2}(x+2y)}{\cancel{2}(5y+2x)}$$

$$\therefore \frac{dy}{dx} = \frac{-(x+2y)}{5y+2x}$$ [1 mark]

$\therefore$ at $(5,-3)$

$$\frac{dy}{dx} = \frac{-(5+2\times -3)}{5\times -3 + 2\times 5}$$

$$= \frac{1}{-5}$$

$$= \frac{-1}{5}$$ [1 mark]

11f

$$t = \tan\left(\frac{x}{2}\right)$$

$$\frac{dt}{dx} = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)$$

$$= \frac{1}{2}\left[1 + \tan^2\left(\frac{x}{2}\right)\right] = \frac{1+t^2}{2}$$

$$\therefore dx = dt \cdot \frac{2}{1+t^2}$$

Note $\sin x = \frac{2t}{1+t^2}$

$\cos x = \frac{1-t^2}{1+t^2}$

$$\therefore \int \frac{1}{4\sin x + 3\cos x}\, dx$$

$$= \int \frac{1}{4 \times \frac{2t}{1+t^2} + \frac{3(1-t^2)}{1+t^2}} \cdot \frac{2}{1+t^2}\, dt \quad [\text{1 mark}]$$

$$= \int \frac{1}{\frac{8t+3-3t^2}{1+t^2}} \cdot \frac{2}{1+t^2}\, dt$$

$$= \int \frac{\cancel{(1+t^2)}}{-3t^2+8t+3} \cdot \frac{2}{\cancel{(1+t^2)}}\, dt$$

$$= 2\int \frac{-1}{(3t+1)(t-3)}\, dt \quad [\text{1 mark}]$$

By Partial fractions

$$\frac{-1}{(3t+1)(t-3)} = \frac{a}{3t+1} + \frac{b}{t-3}$$

$$-1 = a(t-3) + b(3t+1)$$

sub $t=3 \Rightarrow -1 = 10b \rightarrow b = \frac{-1}{10}$

sub $t = \frac{-1}{3} \rightarrow -1 = \frac{-10}{3}a \rightarrow a = \frac{3}{10}$

PTO →

$$= \cancel{2} \int \left[\frac{3}{\cancel{10}_{5}(3t+1)} - \frac{1}{\cancel{10}_{5}(t-3)} \right] dt$$

$$= \frac{1}{5} \int \frac{3}{3t+1}\, dt - \frac{1}{5} \int \frac{1}{t-3}\, dt$$

$$= \frac{1}{5} \left\{ \ln|3t+1| - \ln|t-3| \right\} + c$$

$$= \frac{1}{5} \ln \left| \frac{3t+1}{t-3} \right| + c \qquad \text{[1 mark]}$$

X2

Q11 - Feedback

a very well done, a few made simple arithmetic errors

b i mostly well done, some didn't sketch argand diagram and got angle incorrect.

ii some wasted time and could have worked out quickly from Argand diagram

c i many didn't convert final answer so angle was between 

and missed out on the mark.

ii very well done

d mostly well done - a few students got into a mess or didn't use the correct fractions...

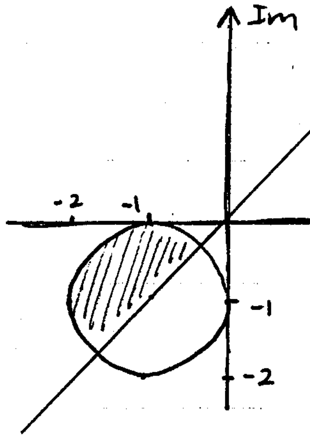
e well done, some made errors rushing and forgot terms or forgot to differentiate expressions.

f mostly well done, those who lost a lot of marks incorrectly worked out $\frac{dt}{dx}$. This is a common X2 question and students should be proficient in doing it quickly & correctly.

Overall - well done, but all students should be aiming for FULL marks in this question.

Q12

a



$|z+1+i| = |z-(-1-i)|$

$\therefore$ circle centre $-1-i$

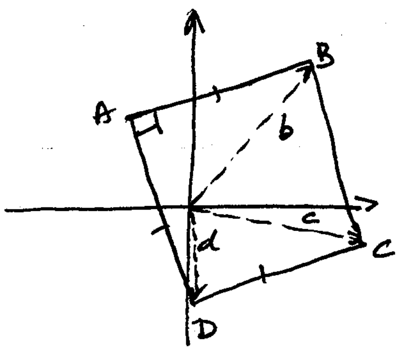
radius 1

for $|z+1+i| \le 1$

[1 mark, for circle & line]

[1 mark, correct shading]

b



via vectors

$CB = b - c$

$CD = d - c$

$CB = (d-c) \times -i$ [1 mark]

$$\left.\begin{aligned} \therefore\ b-c &= -di + ci \\ b &= c + ci - id \\ \therefore\ b &= c(1+i) - id \end{aligned}\right\} \text{[1 mark]}$$

12c

i $\quad z^4+z^2-6$

$= (z^2+3)(z^2-2)$

$= (z^2+3)(z-\sqrt{2})(z+\sqrt{2})$ [1 mark]

ii $\quad (z-\sqrt{3}i)(z+\sqrt{3}i)(z-\sqrt{2})(z+\sqrt{2})$

$\therefore$ roots are $\pm\sqrt{3}i, \pm\sqrt{2}$ [1 mark]

12d

i $\quad \dfrac{(x+4)(x+3)}{x-1} = \dfrac{x^2+7x+12}{x-1}$

$$\begin{array}{r|l} & x+8 \\ \hline x-1 & x^2+7x+12 \\ & x^2-x \\ \hline & 8x+12 \\ & 8x-8 \\ \hline & 20 \end{array}$$

$\therefore \dfrac{(x+4)(x+3)}{(x-1)} = x+8+\dfrac{20}{x-1}$ [1 mark for correct working]

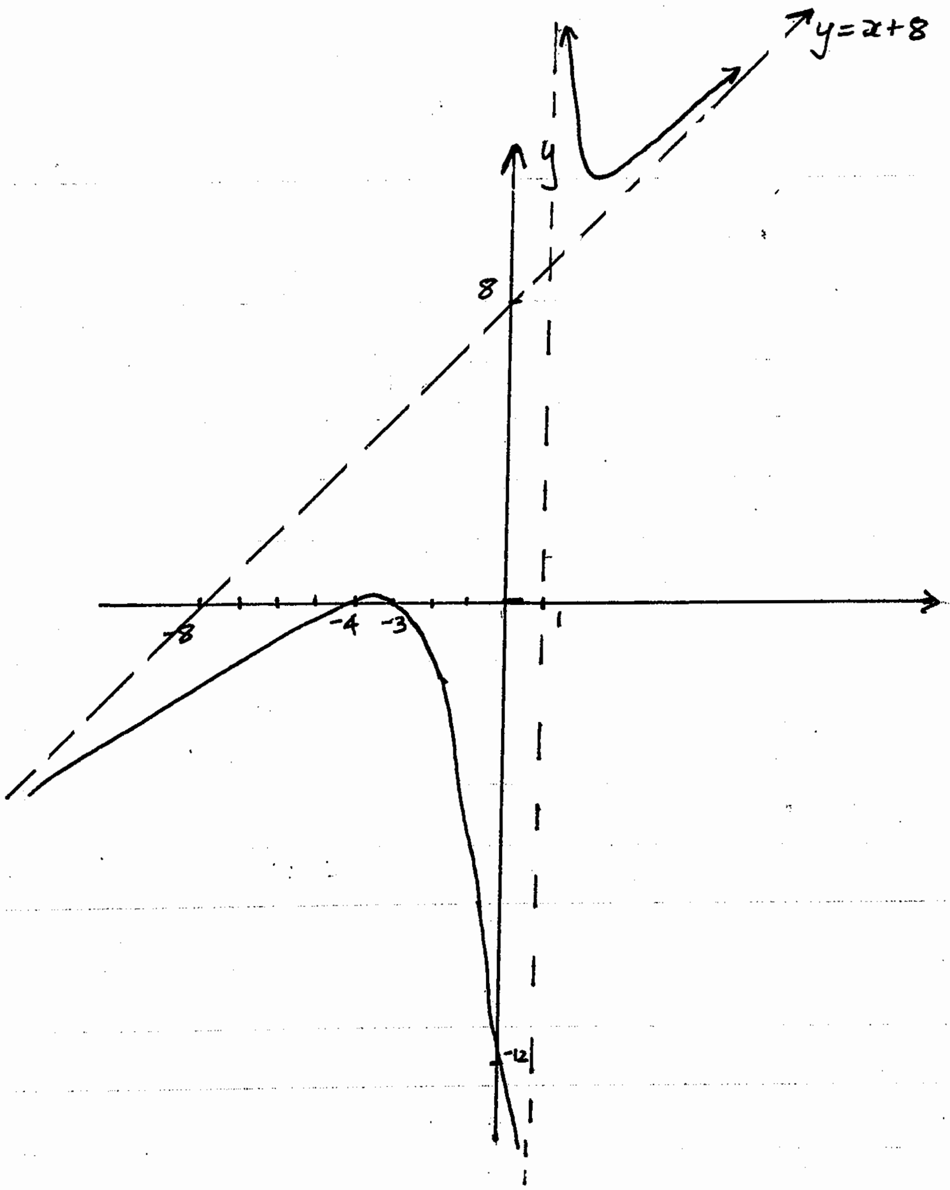
as $x \to \infty$ $\dfrac{20}{x-1} \to 0$

$\therefore$ equation of oblique asymptote is $y=x+8$ [1 mark]

ii

when $y=0 \to$ x-intercepts $x=-4,-3$

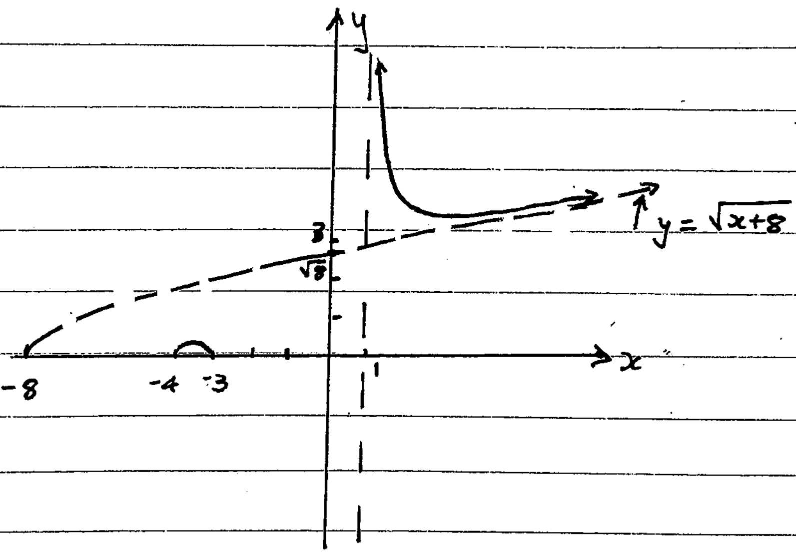
$x=0 \to$ y-intercepts $y=\frac{12}{-1}=-12$



1 mark [intercepts & asymptotes]

1 mark [curve]

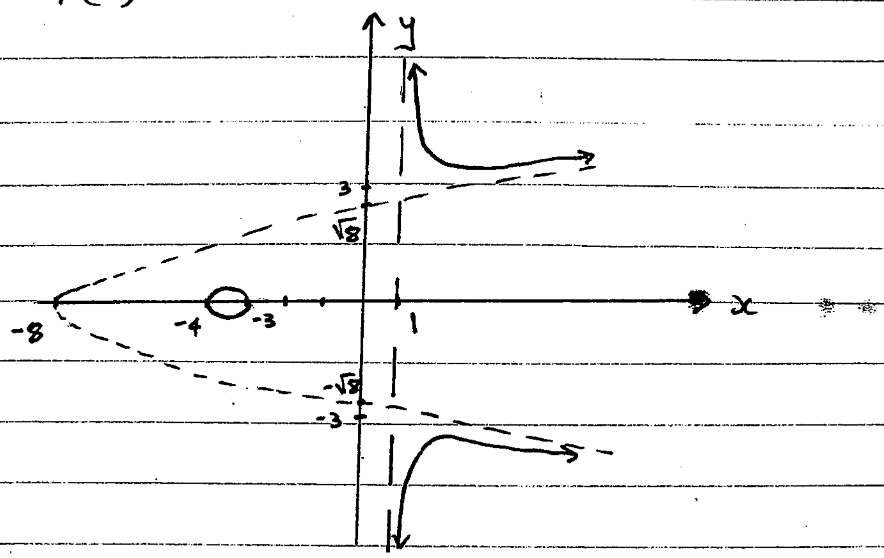
iii $y = \sqrt{f(x)}$



[1 mark, shows $y = \sqrt{x+8}$]

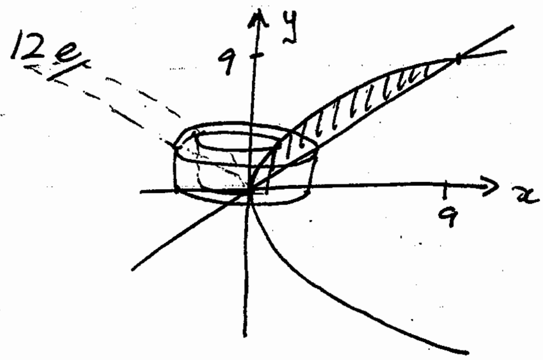
[1 mark, curve]

iv $y^2 = f(x)$



[1 mark] for curve

12 e)



Note: $y^2 = 9x$

and $y = x$

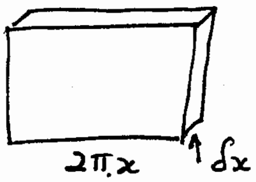
$\therefore x^2 = 9x$

$x^2 - 9x = 0$

$x(x-9) = 0$

$\therefore x = 0, 9$, terminals

$y = \sqrt{9x} - x$



$$V = \sum_{x=0}^{9} 2\pi x(\sqrt{9x} - x) \cdot \delta x \quad \text{as } \delta x \to 0$$

$$\therefore V = \int_0^9 2\pi x(\sqrt{9x} - x)\, dx$$

← mark deducted for NO Σ notation

1 mark

$$V = 2\pi \int_0^9 x(\sqrt{9x} - x)\, dx$$

$$= 2\pi \int_0^9 \left(3x^{3/2} - x^2\right) dx$$

$$= 2\pi \left[\frac{2}{5} \times 3x^{5/2} - \frac{x^3}{3}\right]_0^9$$

$$= 2\pi \left[\frac{6}{5}x^{5/2} - \frac{x^3}{3}\right]_0^9$$

$$= 2\pi \left(\frac{1458}{5} - 243\right)$$

$$= \frac{486\pi}{5} \text{ units}^3$$ [1 mark]

Q12 feedback

a mostly well done, some drew the circle with the incorrect centre, some did incomplete shading

b students either left this blank (a few who need to revise vectors) or did very well. Most got full marks with a variety of methods being used.

c i mostly well done → some didn't factorise z^2-2 !!

ii many didn't read the Q'n, or failed to realise that $\pm\sqrt{2}$ are also complex roots.
Some did a great amount of work and misread the Q'n.

d i very well done

ii most did well, some did multiple graphs on the one axes with very poor labelling...

iii about half of the students didn't show or didn't alter the asymptote to $y=\sqrt{x+8}$

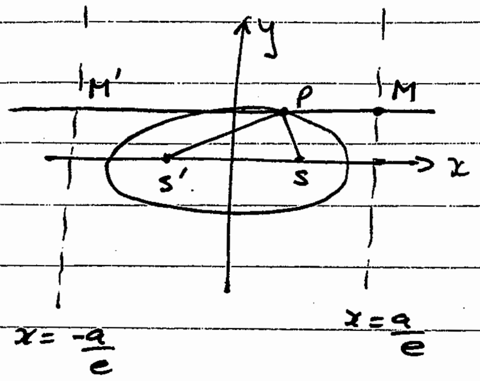
iv was in the main very well done.

e Some didn't realise it was dx (a few, who must revise cylindrical shells). A number lost a mark for not showing Σ as $\delta x \to 0$, then $\int$

- this was mostly well done.

overall – most students should have been able to achieve FULL marks, these were pretty standard HSC questions.

13 a



$$\frac{PS}{PM} = e$$

$$PS = ePM$$

similarly $\quad PS' = ePM'$

$$\therefore \quad PS + PS' = e(PM + PM') \qquad [1 \text{ mark}]$$

we know $PM + PM' = \dfrac{2a}{e}$

$$\therefore \quad PS + PS' = e \times \frac{2a}{e}$$

$$\therefore \quad PS + PS' = 2a \qquad [1 \text{ mark}]$$

b i $\frac{SP}{PM} = e$

as A and A' belong to the ellipse

$\frac{SA}{AN} = e, \quad \frac{SA'}{A'N} = e$

$\therefore SA = e \times AN \quad (1)$

$SA' = e \times A'N \quad (2)$

1 mark

(1) + (2)

$SA + SA' = e(AN + A'N)$ (AA' = SA + SA' from diagram)

$AA' = e(AN + A'N)$

$2a = e(AA' + 2AN)$

$2a = e(2AO + 2AN)$

$2a = 2e(AO + AN)$

$a = e(AO + AN)$

$a = e(ON)$

$\therefore ON = \frac{a}{e}$

1 mark

$\therefore$ positive directrix is $x = \frac{a}{e}$

ii (2) − (1)

$SA' - SA = eA'N - eAN$

$2 \times OS = e(A'N - AN)$

$2 \times OS = e(AA')$

$2 \times OS = e \times 2a$

$\therefore OS = ae$ $\quad \therefore$ focus S is $(ae, 0)$

[1 mark for correct working]

iii $\quad \frac{SP}{PM} = e \qquad S(ae, 0), \; P(x, y), \; M\left(\frac{a}{e}, y\right)$

$\hookrightarrow SP^2 = e^2 PM^2$

$$y^2 + (x - ae)^2 = e^2\left(\frac{a}{e} - x\right)^2$$

$$x^2 - 2aex + a^2e^2 + y^2 = e^2\left(\frac{a^2}{e^2} - \frac{2ax}{e} + x^2\right)$$

$$x^2 \cancel{-2aex} + a^2e^2 + y^2 = a^2 \cancel{-2aex} + e^2x^2$$

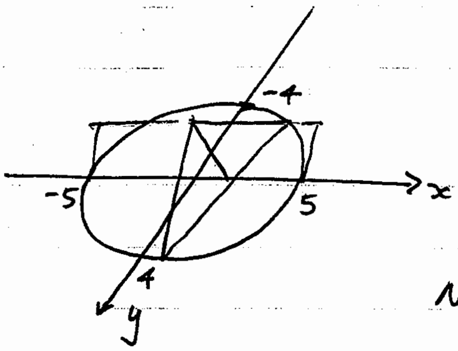
$$x^2 - e^2x^2 + y^2 = a^2 - a^2e^2 \qquad \text{[1 mark]}$$

$$\left[x^2(1-e^2) + y^2 = a^2(1-e^2)\right] \div (1-e^2)$$

$$\left[x^2 + \frac{y^2}{1-e^2} = a^2\right] \div a^2$$

$$\left.\begin{array}{l} \Rightarrow \dfrac{x^2}{a^2} + \dfrac{y^2}{a^2(1-e^2)} = 1 \\ \Rightarrow \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 \qquad \text{where } b^2 = a^2(1-e^2) \end{array}\right\} \text{1 mark}$$

c



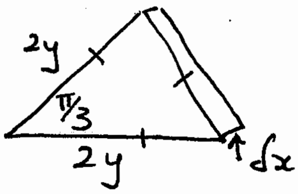
Note: ellipse equation

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

$$\frac{y^2}{16} = 1 - \frac{x^2}{25}$$

$$\frac{y^2}{16} = \frac{1}{25}(25 - x^2)$$

$$y^2 = \frac{16}{25}(25 - x^2) \quad \text{[1 mark]}$$



$$\left.\begin{aligned} \delta V &= \tfrac{1}{2} ab \cdot \sin C \cdot \delta x \\ &= \tfrac{1}{2} \cdot 2y \cdot 2y \cdot \sin \tfrac{\pi}{3} \cdot \delta x \\ &= 2y^2 \cdot \tfrac{\sqrt{3}}{2} \cdot \delta x \end{aligned}\right\} \text{1 mark.}$$

$$\delta V = \sqrt{3}\, y^2 \, \delta x$$

$$V = \sum_{x=-5}^{5} \sqrt{3} \cdot \frac{16}{25}(25 - x^2)\, \delta x \quad \text{as } \delta x \to 0$$ ⇐ mark deducted for NO Σ notation

$$\therefore V = 2 \times \frac{16\sqrt{3}}{25} \int_0^5 (25 - x^2)\, dx$$ 1 mark

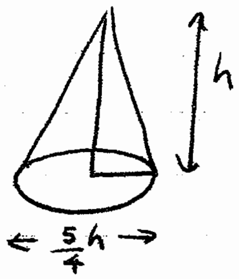
$$= \frac{32\sqrt{3}}{25}\left[25x - \frac{x^3}{3}\right]_0^5$$

$$= \frac{32\sqrt{3}}{25}\left(125 - \frac{125}{3}\right) = \frac{32\sqrt{3}}{\cancel{25}} \times \frac{\cancel{250}^{10}}{3} = \frac{320\sqrt{3}}{3} \text{ units}^3$$

[1 mark]

d

i



i $V = \frac{1}{3}\pi r^2 h$

$r = \frac{5}{4}h \div 2 = \frac{5}{8}h$

$$V = \frac{1}{3} \times \pi \times \left(\frac{5}{8}h\right)^2 \times h$$

$$V = \frac{25\pi h^3}{192}$$ [1 mark, correct working]

ii $\frac{dV}{dh} = \frac{dV}{dt} \times \frac{dt}{dh}$

Note: $\frac{dV}{dh} = \frac{75\pi h^2}{192}$

$= \frac{25}{64}\pi h^2$

$$\frac{25\pi h^2}{64} = 12 \times \frac{dt}{dh}$$ [1 mark]

$$\therefore \frac{dt}{dh} = \frac{25\pi h^2}{768}$$

$$\therefore \frac{dh}{dt} = \frac{768}{25\pi h^2}$$ [1 mark]

sub $h = 2$

$$\frac{dh}{dt} = \frac{768}{25 \times \pi \times 2^2} = \frac{192}{25\pi} \text{ m/min}$$ [1 mark]

decimal answers <u>not</u> accepted

Feedback

Q13 a most got this out, sadly some didn't try and hadn't revised conics. This is a proof that was in our class notes.

b i about half the class struggled with this, many successful students leveraged part a.
Again, a proof that was in our notes.
some didn't try...

ii most got this out, by leveraging b,
again some didn't try... - it was in our notes...

iii this is a very standard proof, many got it out.
some didn't attempt! or didn't start correctly
even though a clear diagram was in front of them.
→ again it was in our notes...

c some got the equation of the ellipse incorrect (most got it o.k)
some didn't draw the 

, again some didn't show Σ notation with $\delta x \to 0$ leading to $\int$ even though we expressly told students to show this in class...

d i most did correctly, sadly some didn't know volume of a cone - or made arithmetic errors.

ii many got full marks
those who didn't had poor working, skipped steps, found reciprocals incorrectly and had poor setting out.
Some didn't give exact answers. In 2u, x1, x2 unless specifically asked for decimal answers please give EXACT answers.

Overall - some students clearly need to spend a lot more time on conics, students need to take more care with their working, especially when they make errors → working can get ECF marks...

Q14 (a) (i) $\displaystyle\int_0^{\pi/6} 2\operatorname{cosec}4x\cdot\tan 2x\,dx$

$$= \int_0^{\pi/6} \frac{2}{2\sin 2x\cdot\cos 2x}\cdot\frac{\sin 2x}{\cos 2x}\,dx$$

$$= \int_0^{\pi/6} \sec^2 2x\,dx$$

1 mark

$$= \frac{1}{2}\Big[\tan 2x\Big]_0^{\pi/6}$$

$$= \frac{\sqrt{3}}{2}$$

Some students did not have the $\frac{1}{2}$ when integrated $\sec^2 2x$.

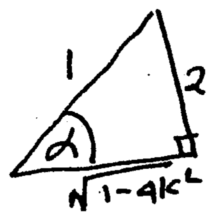
Correct answer 1 mark

(ii) $\displaystyle\int_0^{k} \frac{x^2}{\sqrt{1-4x^2}}\,dx$

let $2x = \sin\theta$
$2\,dx = \cos\theta\,d\theta$
$x=0,\ \theta=0$
$x=k,\ \theta=\alpha$

$$= \int_0^{\alpha} \frac{\frac{1}{4}\sin^2\theta}{\cos\theta}\,\frac{\cos\theta\,d\theta}{2}$$

1 mark



$$= \frac{1}{8}\int_0^{\alpha} \sin^2\theta\,d\theta$$

$$= \frac{1}{16}\int_0^{\alpha} 1-\cos 2\theta\,d\theta$$

1 mark

$$= \frac{1}{16}\Big[\theta - \frac{1}{2}\sin 2\theta\Big]_0^{\alpha}$$

$$= \frac{1}{16}\Big[\alpha - \frac{1}{2}\sin 2\alpha\Big]$$

1 mark

$$= \frac{1}{16}\Big[\alpha - \sin\alpha\cos\alpha\Big]$$

$$= \frac{1}{16}\Big[\sin^{-1}2k - 2k\sqrt{1-4k^2}\Big]$$

Many students got this correct

Q14 (b) $x^2+px+q=0$ roots α and β

$S_n=\alpha^n+\beta^n$, $S_{2n}=S_n^2-2q^n$, $S_{2n+1}=S_n \cdot S_{n+1}+pq^n$

$$S_{2n+1}=S_n \cdot S_{n+1}+pq^n$$

$n=3$ $\quad S_7=S_3 \cdot S_4+pq^3$ — (1) — 1 mark

$n=1$ $\quad S_3=S_1S_2+pq$

$$=-p(p^2-2q)+pq$$

$$S_3=-p^3+3pq \quad \text{— (2)}$$

$$S_{2n}=S_n^2-2q^n$$

$n=2$ $\quad S_4=S_2^2-2q^2$

$$=(p^2-2q)^2-2q^2$$

$$S_4=p^4-4p^2q+2q^2 \quad \text{— (3)}$$

1 mark

Sub in (2) and (3) for S_3 and ~~S_4~~ into (1)

$$S_7=(-p^3+3pq)\cdot(p^4-4p^2q+2q^2)+pq^3$$

$$=-p^7+7p^5q-14p^3q^2+7pq^3$$

→ correct answer 1 mark

Many students identified $S_7=S_3 \cdot S_4+pq^3$, but had difficulties in using the other given information and ~~sq~~ simplifying.

Q14 (c)

(i) $$\frac{5-x}{(2x+3)(x^2+1)} \equiv \frac{a}{2x+3} - \frac{bx-1}{x^2+1} \quad (x \neq -3/2)$$

multiplying both sides by $(2x+3)(x^2+1)$

$$5-x \equiv a(x^2+1) - (bx-1)(2x+3)$$
$$\equiv (a-2b)x^2 + (2-3b)x + (a+3)$$

Matching the constant terms,
$$5 = a+3 \Rightarrow \boxed{a=2}$$ 1 mark

Matching the coefficient of x^2;
$$0 = a-2b \Rightarrow \boxed{b=1}$$ 1 mark

~~Matching the coeffic~~

$$\therefore \frac{5-x}{(2x+3)(x^2+1)} \equiv \frac{2}{2x+3} - \frac{x-1}{x^2+1}$$

(ii) $$\int \frac{5-x}{(2x+3)(x^2+1)}\,dx = \int \frac{2}{2x+3} - \frac{x-1}{x^2+1}\,dx$$
$$= \int \frac{2}{2x+3} - \frac{\frac{1}{2}(2x)}{x^2+1} + \frac{1}{x^2+1}\,dx$$ 1 mark

$$= \ln|2x+3| - \frac{1}{2}\ln(x^2+1) + \tan^{-1}x + C$$ 1 mark

$$= \ln\left(\frac{|2x+3|}{\sqrt{x^2+1}}\right) + \tan^{-1}x + C$$

Many students got this correct.

14(d)

$$\cos(2nx) = \cos\left[(2n-2)x + 2x\right]$$
$$= \cos(2n-2)x\cos 2x - \sin(2n-2)x\sin 2x$$
$$= \cos(2n-2)x\left[2\cos^2 x - 1\right] - 2\sin(2n-2)x\sin x\cos x$$

1 mark

$$= 2\left[\cos(2n-2)x\cos x - \sin(2n-2)x\sin x\right]\cos x - \cos(2n-2)x$$
$$= 2\cos(2n-1)x\cos x - \cos(2n-2)x$$

1 mark

$$\therefore \frac{\cos 2nx}{\cos x} = 2\cos(2n-1)x - \frac{\cos 2(n-1)x}{\cos x}$$

$$\therefore I_n = \int \frac{\cos(2nx)}{\cos x}\,dx = \int 2\cos(2n-1)x - \frac{\cos 2(n-1)x}{\cos x}\,dx$$

1 mark

$$= \frac{2\sin(2n-1)x}{2n-1} - I_{n-1}$$

---

alternate method

$$I_n + I_{n-1} = \int \frac{\cos 2nx}{\cos x} + \frac{\cos 2(n-1)x}{\cos x}\,dx$$
$$= \int \frac{2\cos(2n-1)x\cos x}{\cos x}\,dx$$
$$= \int 2\cos(2n-1)x\,dx$$
$$= \frac{2}{(2n-1)}\sin(2n-1)x$$

$$\Rightarrow I_n = \frac{2\sin(2n-1)x}{(2n-1)} - I_{n-1}$$

* A few students used this alternate method and got the correct result
* Many students split the $\cos(2nx)$ into $\cos\left([2n-1]x + x\right)$ and couldn't go further (except 2 students)

Q15 (a)

(i) $y = \sin^{-1} x + \cos^{-1} x$ — 1 mark

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} + \frac{-1}{\sqrt{1-x^2}}$

$= 0$

(ii) let $A = \cos^{-1}(-x)$

$\Rightarrow -x = \cos A$

$x = -\cos A$

$x = \cos(\pi - A)$

$\pi - A = \cos^{-1} x$

$\Rightarrow \pi - \cos^{-1}(-x) = \cos^{-1}(x)$

$\Rightarrow \cos^{-1}(x) + \cos^{-1}(-x) = \pi$

Could use the graph of $\cos^{-1} x$

Many students could not prove this. — 1 mark

(iii)

$\sin^{-1}(x) + \tan^{-1}\left(\frac{5x}{2x^2+2}\right) = \cos^{-1}(-x) - \frac{\pi}{4}$

$= \pi - \cos^{-1}(x) - \frac{\pi}{4}$ (from (ii)) — 1 mark

$\Rightarrow \tan^{-1}\left(\frac{5x}{2x^2+2}\right) = \pi - \frac{\pi}{4} - (\sin^{-1} x + \cos^{-1} x)$

$= \pi - \frac{\pi}{4} - \frac{\pi}{2}$

$= \frac{\pi}{4}$

$\Rightarrow \frac{5x}{2x^2+2} = 1$

$\Rightarrow 2x^2 - 5x + 2 = 0$

$(2x-1)(x-2) = 0$

$x = \frac{1}{2}$ or $x = 2$

$x = 2$ is not acceptable as for real value of $\cos^{-1} x$ and $\sin^{-1} x$, $-1 \le x \le 1$

$\Rightarrow x = \frac{1}{2}$

many did not see this restriction on the values of x

1 mark with reasoning $x \neq 2$

Q15 (b)

(i) $f(x) = x - \ln(1 + x + \frac{x^2}{2})$ — 1 mark

$f'(x) = 1 - \frac{2(1+x)}{x^2+2x+2}$

$= \frac{x^2}{x^2+2x+2}$

$= \frac{x^2}{(x+1)^2+1}$ — 1 mark

> 0 for all $x \neq 0$

$\Rightarrow f(x)$ is an increasing function

(ii) for $x > 0$ $f(x)$ is an increasing function

when $x = 0$ $f(x) = 0$ — 1 mark

$\Rightarrow f(x) > 0$ for all $x > 0$

$\Rightarrow x - \ln(1 + x + \frac{x^2}{2}) > 0$ — 1 mark

$\Rightarrow e^x > 1 + x + \frac{x^2}{2}$

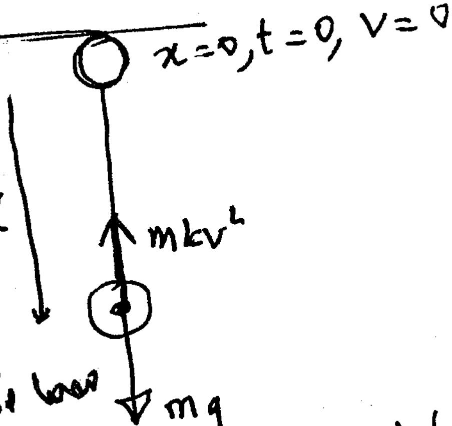
In part (ii) it is important to show that $f(0) = 0$ and since $f(x)$ is an increasing function $f(x) > 0$ for all $x > 0$

Some students missed this in their reasoning

Q15(c)

(i)

It is important to show the force diagram and the direction in which you are applying the Newton's 2nd law



Applying Newton's second Law,

$$mg - mkv^2 = m\ddot{x}$$

$$\Rightarrow \ddot{x} = g - kv^2$$

1 mark

It is important to say where the eqn comes from, i.e. state Newton's 2nd law.

(ii) When the particle reaches its terminal velocity $V\,ms^{-1}$, $\ddot{x}=0$ (or there is no net force acting on the particle)

$$\ddot{x} = g - kv^2$$

when $\ddot{x}=0$, $v=V$

$$0 = g - kV^2$$

$$V = \sqrt{g/k}\ \ ms^{-1}$$

1 mark

(iii)

$$\frac{dv}{dt} = g - kv^2$$

$$= k\left(\frac{g}{k} - v^2\right)$$

$$= k(V^2 - v^2)$$

$$\frac{dt}{dv} = \frac{1}{k(V^2-v^2)}$$

1 mark

Integrating both sides w.r.t v,

$$\int_0^t \frac{dt}{dv}\,dv = \frac{1}{k}\int_0^v \frac{1}{V^2-v^2}\,dv$$

$$t = \frac{1}{2kV}\int_0^v \frac{1}{V-v} + \frac{1}{V+v}\,dv$$

1 mark

$$= \frac{1}{2kV}\ln\left[\frac{V+v}{V-v}\right]$$

$$\Rightarrow \frac{V+v}{V-v} = e^{2kVt}$$

$$\Rightarrow v = V\left[\frac{e^{2kVt}-1}{e^{2kVt}+1}\right]$$

1 mark

(IV) $\ddot{x} = g - kv^2$

$v\dfrac{dv}{dx} = g - kv^2$

$\dfrac{1}{v}\dfrac{dx}{dv} = \dfrac{1}{g-kv^2}$

~~Int~~ $\dfrac{dx}{dv} = \dfrac{v}{g-kv^2}$

Integrating w.r.t v.

$\displaystyle\int_0^x \frac{dx}{dv}\,dv = \int_0^v \frac{v}{g-kv^2}\,dv$

1 mark

$x = -\dfrac{1}{2k}\left[\ln|g-kv^2|\right]_0^v$

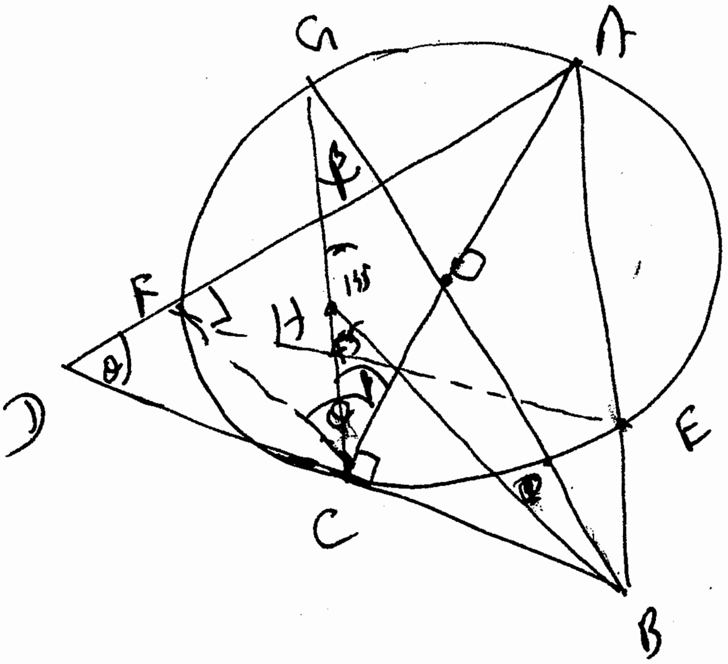
$= -\dfrac{1}{2k}\left[\ln|g-kv^2| - \ln g\right]$

$= \dfrac{1}{2k}\ln\left|\dfrac{g}{g-kv^2}\right|$

1 mark

Many students got this correct.

16 (a)



This question was done poorly
Many did not attempt

(i) Join FC and FE

Let $\angle FCA = \theta$

$\therefore \angle CAF = 90 - \theta$ (Since AC is a diameter, $\angle AFC = 90°$)

But $\angle CDF + \angle CAF = 90°$ ($AC \perp DC$ as DC is a tangent)

$\Rightarrow \angle CDF = \theta$

$\angle AEF = \angle FCA = \theta$ (angles subtended by same chord AF)

2 marks

$\Rightarrow \angle CDF = \angle AEF$

$\therefore$ DFEB is a cyclic quadrilateral as $\angle CDF = \angle AEF \Rightarrow$ exterior $\angle$ = interior opp $\angle$)

1 mark

(ii) $\angle BHC + \angle GHB = 180°$ (st. angle)

$\angle BHC + 3\angle BHC = 180$

$\Rightarrow \angle BHC = 45°$

Let $\angle CGB = \beta$ $\therefore \angle GBH = 45° - \beta$ (Ext $\angle$ = sum of int. opp in ΔGHB)

1 mark

$\angle OCG = \beta$ (OG = OC radii)

$\therefore \angle HCB = 90 + \beta$

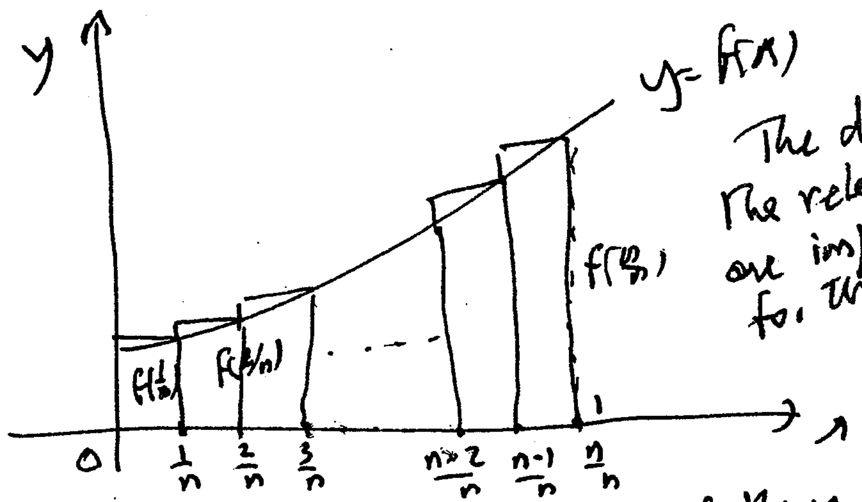
In ΔHCB, $\angle CBH = 180° - \angle BHC - \angle HCB$
$= 180 - 45° - (90 + \beta)$
$= (45 - \beta)$

1 mark

$\Rightarrow \angle GBH = \angle CBH = (45 - \beta)$

Q16 (b)

(i)



The diagram and the relevant statements are important for this part

for diagram → 1 mark

Let A_n be the sum of the areas of the n rectangles approximating the area under the curve $y = f(x)$ from $x = 0$ to $x = 1$. — 1 mark

Note that

$$A_n = \frac{1}{n} f\left(\frac{1}{n}\right) + \frac{1}{n} f\left(\frac{2}{n}\right) + \cdots + \frac{1}{n} f\left(\frac{n}{n}\right)$$

$$= \frac{1}{n}\left\{ f\left(\frac{1}{n}\right) + f\left(\frac{2}{n}\right) + \cdots + f\left(\frac{n}{n}\right)\right\}$$

Therefore $\lim_{n\to\infty} A_n = \int_0^1 f(x)\,dx$ — 1 marks

(ie) $\lim_{n\to\infty} \frac{1}{n}\left\{ f\left(\frac{1}{n}\right) + f\left(\frac{2}{n}\right) + \cdots + f\left(\frac{n}{n}\right)\right\} = \int_0^1 f(x)\,dx$.

(ii) setting $f(x) = \sqrt[3]{x}$, we have

$$\lim_{n\to\infty} \frac{1}{n}\left\{ \frac{\sqrt[3]{1} + \sqrt[3]{2} + \cdots + \sqrt[3]{n}}{\sqrt[3]{n}} \right\}$$

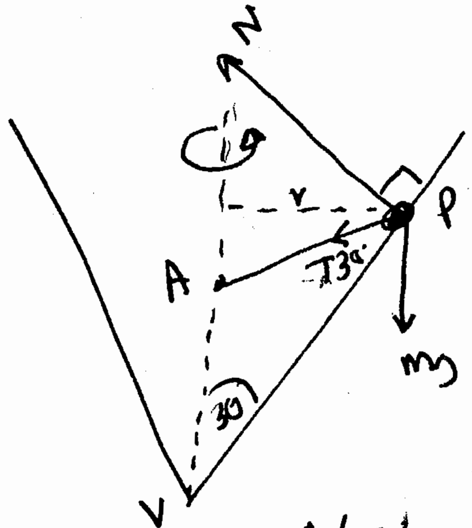
$$= \lim_{n\to\infty} \frac{1}{n}\left\{ \sqrt[3]{\frac{1}{n}} + \sqrt[3]{\frac{2}{n}} + \cdots + \sqrt[3]{\frac{n}{n}} \right\}$$ — 1 mark

$$= \int_0^1 \sqrt[3]{x}\,dx$$

$$= \frac{3}{4}\left[x^{4/3}\right]_0^1$$

$$= \frac{3}{4} \text{ units}^2$$ — 1 mark

Q16 (c)



- Normal reaction N was not considered in some students working
- Students had difficulties with the signs of T and N in the equations

(i) Applying Newton's 2nd Law

$$T\cos 30^\circ + N\cos 30^\circ = \frac{mv^2}{r}$$

$$\Rightarrow \sqrt{3}T + \sqrt{3}N = \frac{2mv^2}{r} \quad (1)$$

Correct Eq^n (1) mark

$$N\sin 30^\circ - T\cos 60^\circ - mg = 0$$

$$\Rightarrow N - T = 2mg \quad (2)$$

Correct Eq^n (1) mark

$$(1) - \sqrt{3}\times(2) \Rightarrow 2\sqrt{3}T = \frac{2mv^2}{r} - 2\sqrt{3}mg$$

$$\Rightarrow T = \frac{m}{\sqrt{3}}\left[\frac{v^2}{r} - \sqrt{3}g\right] \text{ newtons}$$

Correct answer (1) mark

(ii) For the string to be taut $T > 0$

This statement (1) mark

$$\Rightarrow \frac{m}{\sqrt{3}}\left[\frac{v^2}{r} - \sqrt{3}g\right] > 0$$

$$\Rightarrow \frac{v^2}{r} - \sqrt{3}g > 0$$

$$\Rightarrow \frac{v^2}{gr} > \sqrt{3}$$

1 mark

(ii) Some students tried to work out N and used $N > 0$, which will not give the required result.
Some students just stated $T > 0$, but did not say why.